Signals Analysis and Communication
Lecture Notes 1

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In these notes we will:

- Define Signal and System with examples
- Look at many contexts in which signals and systems arise
- Define Continuous Time (CT) and Discrete Time (DT) signals
- Define Signal Energy and Power

1 Signal

The signal is function of one or more independent variables, which contain information about the behavior or nature of some phenomenon. Denoted as:

- $x(t)$ - function of one independent variable, t.
- $x(t_1, t_2, ..., t_N)$ - function of 'N' independent variables, $t_1, ..., t_N$.

2 System

The system responds to or processes one or more input signals producing other signals or some desired behavior, Fig. 1.

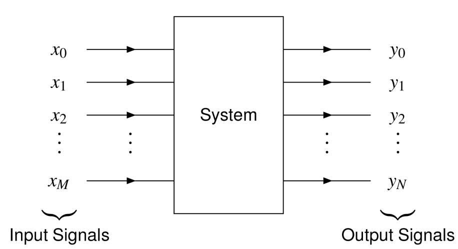


Fig. 1: System

3 Examples

3.1 Electrical Circuit

Fig. 2:

- Signals: Current $x(t)$ and voltage $y(t)$
- System: Electrical Circuit comprising resistor and capacitor connected in series

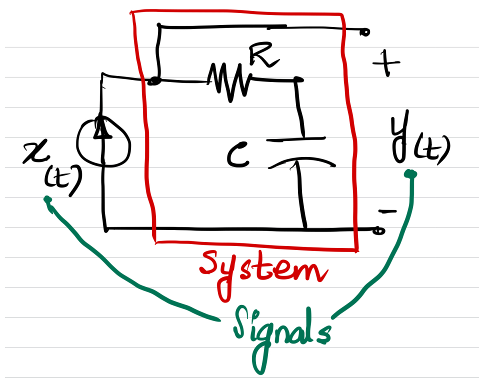


Fig. 2: Example 1 - Electrical Circuit

3.2 Automobile

Fig. 3:

- Signals: Pressure on accelerator and speed
- System: Automobile

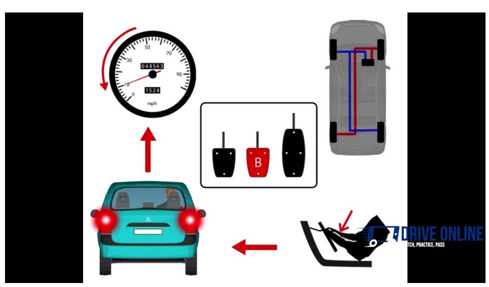


Fig. 3: Example 2 - Automobile

3.3 Robot Arm

- Signals: Control inputs and arm movements
- System: Robot arm

3.4 A Computer Program for automated diagnosis of heart diseases

- Signals: Digitized electrocardiogram (ECG) input and heart rate
- System: A Computer program for automated diagnosis of heart diseases

4 Signals and Systems - Cases of Study

In the many contexts in which signals and systems arise, there are a variety of problems and questions that are of importance.

4.1 Analysis

We are presented with a specific system and are interested in characterizing it in detail to understand how it will respond to various inputs.

4.1.1 Examples

- The analysis of a circuit in order to quantify its response to different voltage and current sources
- The determination of an aircraft's response characteristics both to pilot commands and to wind gusts

4.2 Design of System

The focus is on designing systems to process signals in particular ways, for example:

- To enhance or restore signals that have been degraded in some way
- To extract specific pieces of information from signals

4.2.1 Examples - Enhance or restore signals

- To design systems that will retain the desired signal and reject (at least approximately) the unwanted signal, i.e., the noise. For example, a pilot communicating (desired signal) with an air traffic control tower, the communication can be degraded by the high level of background noise (unwanted) in the cockpit.
- Image restoration of the images from earth-observing satellites
- Image enhancement such as lines (corresponding, for example, to river beds or faults) or regional boundaries in which there are sharp contrasts in color or darkness.

4.2.2 Example - Extract specific information

- The estimation of heart rate from an electrocardiogram

4.3 Design of Signals

The focus is on the design of signals with particular properties.

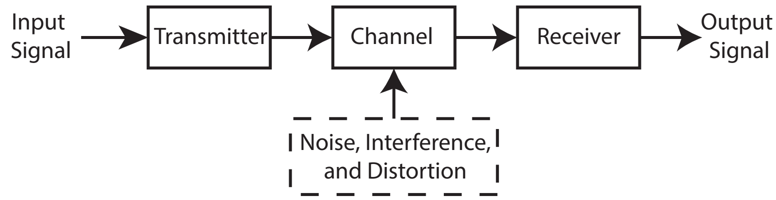


Fig. 4: Communication System

4.3.1 Example

- In communications systems, Fig. 4:
  - Designing signals to meet the constraints and requirements for successful transmission. For example, long distance communication through the atmosphere requires the use of signals with frequencies in a particular part of the electromagnetic spectrum.
  - The design of communication signals must also take into account the need for reliable reception in the presence of both distortion due to transmission through the atmosphere and interference from other signals being transmitted simultaneously by other users

4.4 Modify/Control

To modify or control the characteristics of a given system, perhaps through the choice of specific input signals or by combining the system with other systems.

- The design of control systems to regulate chemical processing plants. Plants of this type are equipped with a variety of sensors that measure physical signals such as temperature, humidity, and chemical composition. The control system in such a plant responds to these sensor signals by adjusting quantities such as flow rates and temperature in order to regulate the ongoing chemical process.
- The design of aircraft autopilots and computer control systems represents another example. In this case, signals measuring aircraft speed, altitude, and heading are used by the aircraft's control system in order to adjust variables such as throttle setting and the position of the rudder and ailerons. These adjustments are made to ensure that the aircraft follows a specified course, to smooth out the aircraft's ride, and to enhance its responsiveness to pilot commands.

5 Continuous Time and Discrete Time Signals

5.1 Continuous Time Signals

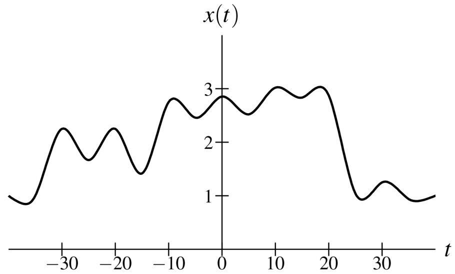
A signal with continuous independent variables is said to be continuous time (CT) (e.g., voltage waveform, velocity).

For CT signals we will enclose the independent variable in parentheses (.).

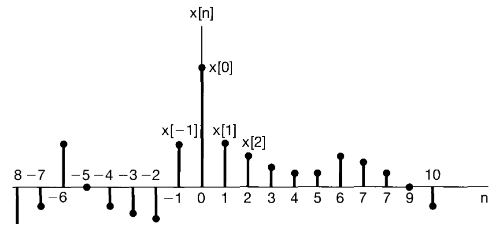
5.2 Discrete Time Signals

A signal with discrete independent variables is said to be discrete time (DT) (e.g., stock market index, computer storage networks).

For DT signals we will enclose the independent variable in brackets [.]. It is important to note that the discrete-time signal $x[n]$ is defined only for integer values of the independent variable.



(a) Continuous Time Signal



(b) Discrete Time Signal

Fig. 5: Continuous Time and Discrete Time Signals

Illustrations of a continuous-time signal $x(t)$ and a discrete-time signal $x[n]$ are shown in Fig. 5.

A very important class of DT signals arises from the sampling of CT signals. In this case, the DT signal $x[n]$ represents successive samples of an underlying phenomenon for which the independent variable is continuous. Because of their speed, computational power, and flexibility, modem digital processors are used to implement many practical systems, ranging from digital autopilots to digital audio systems.

6 Signal Energy and Power

In many, but not all, applications, the signals we consider are directly related to physical quantities capturing power and energy in a physical system.

For example, if $v(t)$ and $i(t)$ are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R} \tag{1}$$

The total energy expended over the time interval $t_1 \leq t \leq t_2$ is

$$E_{[t_1,t_2]} = \int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} \frac{v^2(t)}{R}\,dt \tag{2}$$

The average power over the time interval $t_1 \leq t \leq t_2$ is

$$P_{[t_1,t_2]} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{v^2(t)}{R}\,dt \tag{3}$$

It is a common and worthwhile convention to use similar terminology for power and energy for any CT signal $x(t)$ or any discrete-time signal $x[n]$.

6.1 Energy and Power for Complex Valued Signals

We will see, as the course proceeds, that we will frequently find it convenient to consider signals that take on complex values.

Energy for CT signal:

$$E_{[t_1,t_2]} = \int_{t_1}^{t_2} |x(t)|^2 \, dt \tag{4}$$

Energy for DT signal:

$$E_{[n_1,n_2]} = \sum_{n=n_1}^{n_2} |x(n)|^2 \tag{5}$$

Average Power for CT signal:

$$P_{[t_1,t_2]} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt \tag{6}$$

Average Power for DT signal:

$$P_{[n_1,n_2]} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x(n)|^2 \tag{7}$$

6.2 Energy and Power over an Infinite Time Interval

In many systems we will be interested in examining power and energy in signals over an infinite time interval, i.e., for $-\infty < t < \infty$ or for $-\infty < n < \infty$.

Energy for CT signal:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{8}$$

Energy for DT signal:

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2 \tag{9}$$

Average Power for CT signal:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{10}$$

Average Power for DT signal:

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \tag{11}$$